Algebraic model of baryon resonances

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Abstract. We discuss recent calculations of electromagnetic form factors and strong decay widths of nucleon and delta resonances. The calculations are done in a collective constituent model of the nucleon, in which the baryons are interpreted as rotations and vibrations of an oblate top.

I INTRODUCTION

The study of the properties of baryon resonances is entering a new era with the forthcoming new and more accurate data from new facilities, such as Jefferson Lab., MAMI, ELSA and Brookhaven. Effective models of baryons which are based on three constituents (qqq) share a common spin-flavor-color structure but differ in their assumptions on the spatial dynamics. For example, quark potential models in nonrelativistic [1] or relativized [2] forms emphasize the single-particle aspects of quark dynamics for which only a few low-lying configurations in the confining potential contribute significantly to the eigenstates of the Hamiltonian. On the other hand, some regularities in the observed spectra, such as linear Regge trajectories and parity doubling, hint that an alternative, collective type of dynamics may play a role in the structure of baryons.

In this contribution we discuss a collective model within the context of an algebraic approach [3] and present some results for electromagnetic form factors [4] and strong decay widths [5].

II COLLECTIVE MODEL OF BARYONS

We consider a collective model in which the baryon resonances are interpreted in terms of rotations and vibrations of the string configuration in Fig. 1. A fit to the 3 and 4 star nucleon and delta resonances gives a r.m.s. deviation of 39 MeV [3]. The corresponding oblate top wave functions are spread

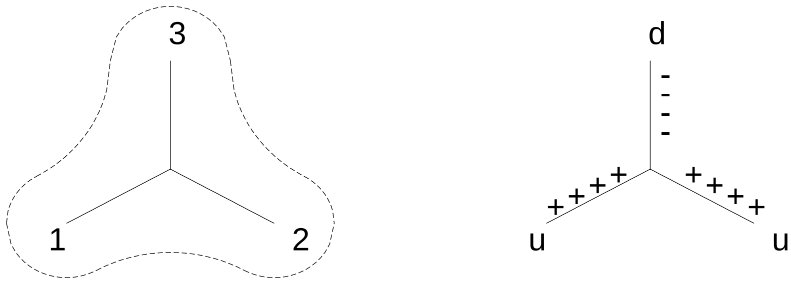

FIGURE 1. Collective model of baryons (the charge distribution of the proton is shown as an example).

over many oscillator shells and hence are truely collective. The baryon wave functions have the form

$$\left| {}^{2S+1}\mathrm{dim}\{SU_f(3)\}_J \left[\mathrm{dim}\{SU_{sf}(6)\}, L^P\right]_{(v_1,v_2);K} \right\rangle \ . \tag{1}$$

The spin-flavor part has the usual $SU_{sf}(6)$ classification and determines the permutation symmetry of the state. The spatial part is characterized by the labels: $(v_1, v_2); K, L^P$, where (v_1, v_2) denotes the vibrations (stretching and bending) of the string configuration in Fig. 1; K denotes the projection of the rotational angular momentum L on the body-fixed symmetry-axis and P the parity. Finally, S and J are the spin and the total angular momentum $\vec{J} = \vec{L} + \vec{S}$. In this contribution we focus on the nucleon resonances which are interpreted as rotational excitations of the $(v_1, v_2) = (0, 0)$ vibrational ground state.

The electromagnetic (strong) coupling is assumed to involve the absorption or emission of a photon (elementary meson) from a single constituent. The collective form factors and decay widths are obtained by folding with a probability distribution for the charge and magnetization along the strings of Fig. 1

$$g(\beta) = \beta^2 \, \mathrm{e}^{-\beta/a}/2a^3 \ , \tag{2}$$

where β is a radial coordinate and a is a scale parameter. In the algebraic approach these form factors and decay widths can be obtained in closed analytic form (in the limit of a large model space) which allows us to do a straightforward and systematic analysis of the experimental data.

The ansatz of Eq. (2) for the probability distribution is made to obtain a dipole form for the proton electric form factor

$$G_E^p(k) = \frac{1}{(1 + k^2 a^2)^2} \ . \tag{3}$$

The same distribution is used to calculate transition form factors and decay widths. As a result, all collective form factors are found to drop as powers of

the momentum transfer [4]. This property is well-known experimentally and is in contrast with harmonic oscillator based quark models in which all form factors fall off exponentially.

III ELECTROPRODUCTION

Electromagnetic inelastic form factors can be measured in electroproduction of baryon resonances. They are expressed in terms of helicity amplitudes A_ν^N, where ν indicates the helicity and N represents proton (p) or neutron (n) couplings. In a string-like model of hadrons one expects [6] on the basis of QCD that strings will elongate (hadrons swell) as their energy increases. This effect can be easily included in the present analysis by making the scale parameters of the strings energy- dependent. We use here the simple ansatz

$$a = a_0\Big(1 + \xi \, \frac{W - M}{M}\Big) \ , \tag{4}$$

where M is the nucleon mass and W the resonance mass. This ansatz introduces a new parameter (ξ), the stretchability of the string.

In Fig. 2 we show the transverse helicity amplitudes A_ν^p for the $N(1520)D_{13}$ resonance calculated in the Breit frame (a factor of $+i$ is suppressed). The scale parameter a is determined in a simultaneous fit to the nucleon charge radii and the nucleon elastic form factors: $a = 0.232$ fm [4]. The effect of stretching on the helicity amplitudes is sizeable (especially if one takes the value $\xi \approx 1$ which is suggested by QCD arguments [6] and the Regge behavior of nucleon resonances). The data for $N(1520)D_{13}$ (and also for $N(1680)F_{15}$) show a clear indication that the form factors are dropping faster than expected on the basis of the dipole form. The disagreement at low momentum transfer may be due to the neglect of the meson cloud.

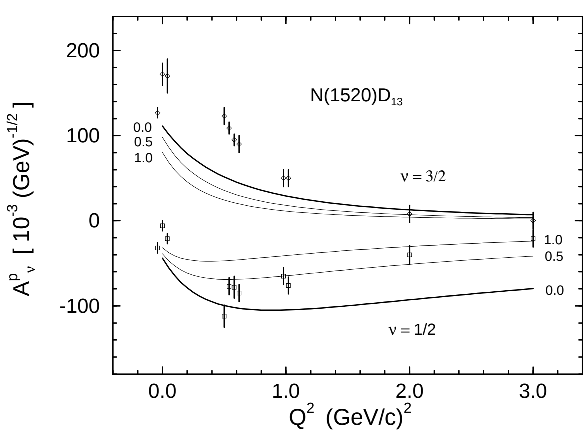

FIGURE 2. Effect of hadron swelling for excitation of $N(1520)D_{13}$. The curves are labelled by the stretching parameter ξ of Eq. (4).

IV STRONG DECAY WIDTHS

In addition to electromagnetic couplings, strong decays provide an important, complementary, tool to study the structure of baryons. We consider decays with emission of π and η. The experimental data [7] are shown in Table 1, where they are compared with the results of our calculation. The calculated values depend on two parameters g and h in the transition operator and on the scale parameter a of Eq. (2). These parameters are determined from a least-square fit to the $N\pi$ partial widths (which are relatively well known) with the exclusion of the S_{11} resonances whose assignments are not clear due to possible mixing between the $N(1535)$ and $N(1650)$ and/or the possible presence of a third S_{11} resonance [8].

The calculation of $N\pi$ decay widths is found to be in fair agreement with experiment (see Table 1). These results are to a large extent a consequence of spin-flavor symmetry. The calculated widths for the $N\eta$ channel are systematically small. We emphasize here that, since the transition operator was determined from the $N\pi$ decays, the η decays are calculated without introducing any further parameters. The results of this analysis suggest that the large η width for the $N(1535)S_{11}$ is not due to a conventional q^3 state. One possible explanation is the presence of another state in the same mass region, *e.g.* a quasi-bound meson-baryon S wave resonance just below or above threshold, for example $N\eta$, $K\Sigma$ or $K\Lambda$ [9]. Another possibility is an exotic configuration of four quarks and one antiquark ($q^4\bar{q}$).

TABLE 1. $N\pi$ and $N\eta$ decay widths of 3 and 4 star nucleon resonances in MeV.

State	Mass	Resonance	$\Gamma(N\pi)$		$\Gamma(N\eta)$	
			th	exp	th	exp
S_{11}	$N(1535)$	$^{2}8_{1/2}[70,1^-]_{(0,0);1}$	85	79 ± 38	0.1	74 ± 39
S_{11}	$N(1650)$	$^{4}8_{1/2}[70,1^-]_{(0,0);1}$	35	130 ± 27	8	11 ± 6
P_{13}	$N(1720)$	$^{2}8_{3/2}[56,2^+]_{(0,0);0}$	31	22 ± 11	0.2	
D_{13}	$N(1520)$	$^{2}8_{3/2}[70,1^-]_{(0,0);1}$	115	67 ± 9	0.6	
D_{13}	$N(1700)$	$^{4}8_{3/2}[70,1^-]_{(0,0);1}$	5	10 ± 7	4	
D_{15}	$N(1675)$	$^{4}8_{5/2}[70,1^-]_{(0,0);1}$	31	72 ± 12	17	
F_{15}	$N(1680)$	$^{2}8_{5/2}[56,2^+]_{(0,0);0}$	41	84 ± 9	0.5	
G_{17}	$N(2190)$	$^{2}8_{7/2}[70,3^-]_{(0,0);1}$	34	67 ± 27	11	
G_{19}	$N(2250)$	$^{4}8_{9/2}[70,3^-]_{(0,0);1}$	7	38 ± 21	9	
H_{19}	$N(2220)$	$^{2}8_{9/2}[56,4^+]_{(0,0);0}$	15	65 ± 28	0.7	
$I_{1,11}$	$N(2600)$	$^{2}8_{11/2}[70,5^-]_{(0,0);1}$	9	49 ± 20	3	

V SUMMARY AND CONCLUSIONS

We have analyzed simultaneously electromagnetic form factors and strong decay widths in a collective model of the nucleon. The helicity amplitudes are folded with a probability distribution for the charge and magnetization, which is determined from the well-established dipole form of the nucleon electromagnetic form factors. The same distribution function is used to calculate the transition form factors for the excited baryons. As a result, all form factors drop as powers of the momentum transfer.

For electromagnetic couplings we find that the inclusion of the stretching of baryons improves the calculation of the helicity amplitudes for large values of the momentum transfer. The disagreement for small values of the momentum transfer $0 \leq Q^2 \leq 1$ (GeV/c)^2 may be due to coupling of the photon to the meson cloud, *i.e.* configurations of the type $q^3 - q\bar{q}$. Since such configurations have much larger spatial extent than q^3, their effects are expected to drop faster with momentum transfer than the constituent form factors.

An analysis of the strong decay widths into the $N\pi$ and $N\eta$ channels shows that the π decays follow the expected pattern. Our calculations do not show any indication for a large η width, as is observed for the $N(1535)S_{11}$ resonance. The observed large η width indicates the presence of another configuration, which is outside the present model space.

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